

# PCAC in Nuclear Medium and the Lovelace - Shapiro - Veneziano formula

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## Abstract

A simple way to enforce the Adler zero condition for pion amplitudes in the nuclear medium is to use the Lovelace quantization condition but with modified Regge Slope  $\alpha'$ . This latter is related to change in the gluon condensate. Increasing nuclear density leads to a relative increase in the Regge Slope  $\alpha'$ . Denoting this increase by a scale factor  $\lambda$ , the drop in the  $\rho$ -mass,  $\Delta - N$  mass difference, increase in  $\pi\pi$  scattering length, decrease of pion decay constant etc. are simply related to  $\lambda$ .

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There is substantial evidence that nucleon properties are modified inside a nucleus. These are commonly the quenching of the magnetic moments and Gamow-Teller matrix elements, Nolen-Schiffer anomaly, nuclear EMC effect etc. The question of plausible pion and kaon condensation, restoration of chiral symmetry are also of great interest. Further recent experiments on heavy ion collisions in which large dilepton cross-sections are observed, have been interpreted as due to “dropping” of vector meson masses in the nuclear medium[1].

Here we shall address some of these questions using the following two principles.

- 1) Even if chiral symmetry is partially restored in the nuclear medium, as long as it remains a spontaneously broken symmetry the pion retains its Nambu-Goldstone character. PCAC then demands that an amplitude with a pion on the external line must vanish when the external pion’s four momentum goes to zero (Adler Zero).
- 2) Hadron masses are determined by the slope  $\alpha'$  of Regge trajectories or the string tension which can be related to the distribution of gluonic fields in the physical vacuum state. The gluonic fields are modified in the nuclear medium and lead to a reduction in the string tension or an effective increase in the value of  $\alpha'$ .

These two principles find an easy expression in the Lovelace-Shapiro-Veneziano[2, 3] formula. Let us briefly recapitulate some of the old analysis,

using currently known experimental information. The amplitude  $\pi^+(p_1) + \pi^-(p_2) \rightarrow \pi^+(p_3) + \pi^-(p_4)$  can be represented by a single term formula[2, 3].

$$A(s, t) = -\beta \frac{\Gamma(1 - \alpha(s))\Gamma(1 - \alpha(t))}{\Gamma(1 - \alpha(s) - \alpha(t))} \quad (1)$$

Here  $\alpha(s)$  is the linear Regge trajectory represented by

$$\alpha(s) = \alpha_0 + \alpha' s \quad (2)$$

and the Mandelstam invariants are

$$s = (p_1 + p_2)^2 \quad t = (p_1 - p_3)^2 \quad u = (p_1 - p_4)^2 \quad (3)$$

The constant  $\beta$  will be determined below. As pointed out by Lovelace[2], setting  $p_{1\mu} = 0$  and demanding that  $A(s, t)$  vanish, leads to the quantization condition

$$\alpha(m_\pi^2) = 1/2 \quad (4)$$

The intercept  $\alpha_0$  and the slope  $\alpha'$  in eqn.(2) can be determined using the spin 1,  $\rho(770)$  and the spin 3,  $\rho_3(1690)$  masses. We have [4].

$$m_\rho = 768.5 \pm 0.6 MeV \quad m_{\rho_3} = 1691 \pm 5 MeV \quad (5)$$

Using  $\alpha(m_\rho^2) = 1$  and  $\alpha(m_{\rho_3}^2) = 3$  and the median mass values we find

$$\alpha_0 = 0.479, \quad \alpha' = 0.881(GeV)^{-2} \quad (6)$$

This then gives

$$\alpha(m_\pi^2) = \alpha_0 + \alpha' m_\pi^2 = 0.497 \quad (7)$$

which is remarkably close to Lovelace's quantization condition eqn.(4).

The s-wave scattering length  $a_0^0$  and  $a_0^2$  can be obtained using definite isospin combinations of the amplitude eqn.(1) as described in ref.[2] and [3]. We find [5]

$$a_0^0 = \frac{\pi\beta\alpha'm_\pi^2}{m_\pi} \cdot \frac{7}{2} \left[ 1 + \frac{20}{7} \ln 2 (\alpha'm_\pi^2) + 22.1 (\alpha'm_\pi^2)^2 + \dots \right] \quad (8)$$

$$a_0^2 = \frac{-\pi\beta\alpha'm_\pi^2}{m_\pi} \left[ 1 - 4 \ln 2 (\alpha'm_\pi^2) + (3.36) (\alpha'm_\pi^2)^2 + \dots \right] \quad (9)$$

The constant  $\beta$  can be determined by relating the I=1 amplitue to the experimental width  $\Gamma$  of the  $\rho$  [3]

$$\beta = \frac{3}{4} \frac{\Gamma m_\rho^2}{q^3} \approx 1.45 \quad (10)$$

Here  $q$  is the C.M. momentum of the decaying pions. Alternately instead of using the experimental  $\rho$ -width one can use the famous KSRF relation[7] to get

$$\beta = \frac{g_{\rho\pi\pi}^2}{8\pi} = \frac{1}{8\pi} \frac{2m_\rho^2}{F_\pi^2} \quad (F_\pi = 184 MeV) \quad (11a)$$

$$\simeq 1.39 \quad (11b)$$

Multiplying eqn.(11a) by  $\alpha'$  and using  $2\alpha'm_\rho^2 = 1$  we can write<sup>1</sup>.

$$\beta\alpha' = \frac{1}{8\pi} \quad \frac{1}{F_\pi^2} \quad (12)$$

which will be useful later.

Before we turn to the nuclear medium, it is useful to recall the generalization of the Lovelace Quantization Condition eqn.(4) by Ademollo, Veneziano and Weinberg[8]. They obtained

$$\alpha_X(0) - \alpha_A(0) = 1/2. \quad (13)$$

where the hadron  $X$  couples to the hadron  $A$  by pion with  $X$  and  $A$  having opposite normality. (Normality = parity  $\otimes(-1)^J$  bosons; parity  $\otimes(-1)^{J-1/2}$  for fermions). Using eqn.(13) they deduced several relations

$$\alpha' (m_{K^*}^2 - m_K^2) = 1/2 \quad \alpha' (m_\Delta^2 - m_N^2) = 1/2 \quad (14)$$

etc. besides of course the Lovelace quantization, condition.

$$\alpha' (m_\rho^2 - m_\pi^2) = 1/2$$

All this predates QCD. Let us now consider the question, how  $\alpha'$  is determined in QCD. Several years ago, Nambu[10] derived a string like equation

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<sup>1</sup>Using eqn.(12) in eqn.(8) and (9) makes the leading coefficients a factor  $\pi/2$  larger than the values given by Weinberg[6]. However if one keeps in eqn.(1) only the  $\alpha(s) = 1$   $\sigma$ -pole term one is lead back to Weinberg's values. See related comments in [8]. For a recent discussion of Chiral perturbation theory for  $\pi\pi$  scattering see[9]. The precise numerical value of  $\beta$  plays no role in Lovelace quantization condition eqn.(4) and in our discussion of PCAC in nuclear medium

for the path ordered phase-factor

$$U[\sigma] = p \exp \left( i \int_{\sigma} A_{\mu} dz^{\mu} \right) \quad (15)$$

where  $\sigma$  is a space-like curve and  $A_{\mu} = g \sum A_{\mu}^a \lambda^a / 2$ .

By considering the variation in  $U[\sigma]$  for a normal displacement and iterating it Nambu obtained the equation

$$\left( \frac{\delta}{\delta \sigma_{\mu t}} \frac{\delta}{\delta \sigma_{\mu t}} + C \right) U[\sigma] = 0 \quad (16)$$

with

$$C = G_{\mu t} G_{\mu t} \quad (17)$$

where  $\mu$  and  $t$  refer to normal and tangential directions along the string and  $G_{\alpha\beta} = \partial_{\alpha} A_{\beta} - \partial_{\beta} A_{\alpha} - i[A_{\alpha}, A_{\beta}]$  is the gluon field tensor, Identifying eqn.(16) with the string equation from the Nambu-Goto Action leads to

$$C = - \left( \frac{1}{2\pi\alpha'} \right)^2 \quad (18)$$

Nambu further pointed out that the energy density of the string regarded as a chromoelectric flux tube is consistent in magnitude with the value of the gluon condensate  $\langle 0 | G_{\alpha\beta} G^{\alpha\beta} | 0 \rangle$  determined from QCD sum rules[11]. Also he estimated the cross-section area of the flux tube  $a$  to be

$$a \approx (0.5 \text{ fermi})^2 \quad (19)$$

The slope of the Regge trajectory  $\alpha'$  can also be determined in the MIT bag model. Johnson and Thorn[12] derived the relation

$$\alpha' = \frac{1}{16\pi^{3/2}} \left(\frac{3}{2}\right)^{1/2} \frac{1}{\sqrt{\alpha_s} \sqrt{B}} \quad (20)$$

which from the phenomenological values  $\alpha_s = 0.5$  and  $B^{1/4} = 146 MeV$  used in Bag model studies yields  $\alpha' = 0.91(GeV)^{-2}$  in agreement with experiment. The Bag pressure  $B$  can be readily interpreted as the difference between the energy density of the physical vacuum (outside of the Bag) and perturbative vacuum (inside of the Bag). Now by the trace anomaly the energy density of the physical vacuum is related to the gluon condensate,  $\langle 0|G_{\mu\nu}G^{\mu\nu}|0 \rangle$ .

Guided by the above considerations we shall assume

$$\frac{1}{(\alpha')^2} = K \langle 0|G_{\mu\nu}G^{\mu\nu}|0 \rangle \quad (21)$$

where the precise value of the constant  $K$  will not be important in the following.

We now turn to the nuclear medium. At low nuclear densities chiral symmetry would remain broken even if its magnitude changes significantly. It follows from eqn.(19) that the transverse string radius is small compared to internucleon separation in a nucleus. Also as emphasized by Migdal[13] the transition from weak coupling to strong coupling in  $\alpha_s$  takes place at small distances 0.2 to 0.3 fm and the string picture makes sense even for low angular momenta. It is then reasonable to extend the dual amplitude to the nuclear case at least for low densities by replacing  $\alpha(s)$  by  $\alpha_{\text{med}}(s)$  where

we write

$$\alpha_{\text{med}}(s) = \alpha_{0, \text{med}} + \alpha'_{\text{med}} \cdot s \quad (22)$$

By virtue of eqn.(21), the modified string tension  $\alpha'_{\text{med}}$  can be obtained from

$$\frac{(\alpha'_{\text{med}})^2}{(\alpha')^2} = \frac{\langle 0|G_{\alpha\beta}G^{\alpha\beta}|0 \rangle}{\langle 0|G_{\alpha\beta}G^{\alpha\beta}|0 \rangle_{\text{med}}} = \lambda^2 \quad (23)$$

where  $\langle 0|G_{\mu\nu}G^{\mu\nu}|0 \rangle_{\text{med}}$  denotes the value of the gluon condensate in the nuclear medium.  $\lambda$  is a convenient scale factor (in the medium  $\lambda > 1$  see below). PCAC or the Adler Zero in medium will now be satisfied by the modified Lovelace quantisation condition

$$\alpha_{\text{med}}(m_\pi^2) = 1/2 \quad (24)$$

Eqn.(23) and eqn.(24) are sufficient to derive a number of results. We indicate a few below. We use an asterisk to denote the in-medium value. From eqn.(4), (14), (23) and (24) we have[14].

$$\frac{(m_\rho^{*2} - m_\pi^2)}{(m_\rho^2 - m_\pi^2)} = \frac{\alpha'}{\alpha'_{\text{med}}} = \frac{1}{\lambda} \quad (25)$$

$$\frac{(m_\Delta^{*2} - m_N^{*2})}{(m_\Delta^2 - m_N^2)} = \frac{1}{\lambda} \quad (26)$$

The  $\Delta - N$  mass difference must decrease within the medium. Using eqn.(8), we find that the S-wave scattering length  $a_0^0$  should increase inside the medium.

$$a_0^{0*} = a_0^0 \lambda \quad (27)$$

From eqn.(12) the pion-decay constant  $F_\pi$  decreases inside the nucleus

$$F_\pi^* = F_\pi \lambda^{-1/2} \quad (28)$$

It is easy to understand Eqn.(25), (27) and (28). In a picture of the pion as a quark antiquark bound state,  $F_\pi^* < F_\pi$  corresponds to decrease of the bound state wave function at the origin or an increase in the radius. Decrease of  $|\psi(0)|^2$  in turn implies a reduction of the hyperfine splitting between the singlet pion and the triplet  $\rho$  and of course an increase in the radius of the pion implies an increase in the  $\pi\pi$  cross section.

The scaling ratio  $\lambda$  can be estimated as follows. One can write in the linear density approximation[15].

$$\langle 0|G_{\mu\nu}G^{\mu\nu}|0 \rangle_{\text{med}} = \langle 0|G_{\mu\nu}G^{\mu\nu}|0 \rangle + \rho \langle N|G_{\mu\nu}G^{\mu\nu}|N \rangle$$

where  $\rho$  is the density of nucleons inside the nucleus ( $\rho \approx 0.16 \text{ fm}^{-3}$ ). The matrix element  $\langle N|G_{\mu\nu}G^{\mu\nu}|N \rangle$  is related to nucleon mass by the trace-anomaly. One has[16]

$$\langle N|G_{\mu\nu}G^{\mu\nu}|N \rangle \approx -32\pi^2 \otimes 78 \text{ MeV}$$

Using the QCD sum rule estimate[11]  $\langle 0|G_{\mu\nu}G^{\mu\nu}|0 \rangle \approx 0.5(\text{GeV})^4$  this means  $\lambda \approx (1.06)^{1/2}$  at ordinary nuclear densities. In a heavy ion collision we can expect  $\lambda$  to be significantly larger than 1.

One can also look for other experimental signatures. For example, the transverse momentum distribution of secondaries in high energy collisions

is exponentially suppressed. This damping factor is proportional to  $(\alpha')^{-1/2}$ . With increasing  $\alpha'_{\text{med}}$ , we should expect less damping i.e., the average transverse momentum of the secondaries increases with density or equivalently centre mass energy of the collision.

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